

Kerr Black Hole

```
Manipulate[

If[
  ! slidersEnabled,
  {pT, aI, rI, iL, θI, pθI, frame, tailLength, zoomManual} = presetValues;

  slidersEnabled = True;
  ];

viewRadius = 10;

viewθ = 0.85 π/2;
viewφ = 0.35 π/2;
divergence = 0.05 π/2;

rightViewPoint =

viewRadius {
Sin:viewθ Cos:viewφ,
Sin:viewθ Sin:viewφ,
Cos:viewθ
};

leftViewPoint =

viewRadius {
Sin:viewθ Cos:[viewφ - divergence],
Sin:viewθ Sin:[viewφ - divergence],
Cos:viewθ
};

Ee =
ε /.
Solve[
(-aI2 pθI2 + 2 iL2 rI +
 2 pθI2 rI - aI2 rI2 -
 iL2 rI2 - pθI2 rI2 +
 2 rI3 - rI4 -
 4 aI iL rI ε + 2 aI2 rI ε2 +
 aI2 rI2 ε2 + rI4 ε2 +
 2 aI3 rI2 ε + aI2 rI3 ε -
 2 aI2 rI2 ε2 - rI4 ε2 -
 2 aI3 rI2 ε2 + aI2 rI3 ε2 +
 2 aI2 rI2 ε3 - rI4 ε3 -
 2 aI3 rI2 ε3 + aI2 rI3 ε3 +
 2 aI2 rI2 ε4 - rI4 ε4 -
 2 aI3 rI2 ε4 + aI2 rI3 ε4 +
 2 aI2 rI2 ε5 - rI4 ε5 -
 2 aI3 rI2 ε5 + aI2 rI3 ε5 +
 2 aI2 rI2 ε6 - rI4 ε6 -
 2 aI3 rI2 ε6 + aI2 rI3 ε6 +
 2 aI2 rI2 ε7 - rI4 ε7 -
 2 aI3 rI2 ε7 + aI2 rI3 ε7 +
 2 aI2 rI2 ε8 - rI4 ε8 -
 2 aI3 rI2 ε8 + aI2 rI3 ε8 +
 2 aI2 rI2 ε9 - rI4 ε9 -
 2 aI3 rI2 ε9 + aI2 rI3 ε9 +
 2 aI2 rI2 ε10 - rI4 ε10 -
 2 aI3 rI2 ε10 + aI2 rI3 ε10 +
 2 aI2 rI2 ε11 - rI4 ε11 -
 2 aI3 rI2 ε11 + aI2 rI3 ε11 +
 2 aI2 rI2 ε12 - rI4 ε12 -
 2 aI3 rI2 ε12 + aI2 rI3 ε12 +
 2 aI2 rI2 ε13 - rI4 ε13 -
 2 aI3 rI2 ε13 + aI2 rI3 ε13 +
 2 aI2 rI2 ε14 - rI4 ε14 -
 2 aI3 rI2 ε14 + aI2 rI3 ε14 +
 2 aI2 rI2 ε15 - rI4 ε15 -
 2 aI3 rI2 ε15 + aI2 rI3 ε15 +
 2 aI2 rI2 ε16 - rI4 ε16 -
 2 aI3 rI2 ε16 + aI2 rI3 ε16 +
 2 aI2 rI2 ε17 - rI4 ε17 -
 2 aI3 rI2 ε17 + aI2 rI3 ε17 +
 2 aI2 rI2 ε18 - rI4 ε18 -
 2 aI3 rI2 ε18 + aI2 rI3 ε18 +
 2 aI2 rI2 ε19 - rI4 ε19 -
 2 aI3 rI2 ε19 + aI2 rI3 ε19 +
 2 aI2 rI2 ε20 - rI4 ε20 -
 2 aI3 rI2 ε20 + aI2 rI3 ε20 +
 2 aI2 rI2 ε21 - rI4 ε21 -
 2 aI3 rI2 ε21 + aI2 rI3 ε21 +
 2 aI2 rI2 ε22 - rI4 ε22 -
 2 aI3 rI2 ε22 + aI2 rI3 ε22 +
 2 aI2 rI2 ε23 - rI4 ε23 -
 2 aI3 rI2 ε23 + aI2 rI3 ε23 +
 2 aI2 rI2 ε24 - rI4 ε24 -
 2 aI3 rI2 ε24 + aI2 rI3 ε24 +
 2 aI2 rI2 ε25 - rI4 ε25 -
 2 aI3 rI2 ε25 + aI2 rI3 ε25 +
 2 aI2 rI2 ε26 - rI4 ε26 -
 2 aI3 rI2 ε26 + aI2 rI3 ε26 +
 2 aI2 rI2 ε27 - rI4 ε27 -
 2 aI3 rI2 ε27 + aI2 rI3 ε27 +
 2 aI2 rI2 ε28 - rI4 ε28 -
 2 aI3 rI2 ε28 + aI2 rI3 ε28 +
 2 aI2 rI2 ε29 - rI4 ε29 -
 2 aI3 rI2 ε29 + aI2 rI3 ε29 +
 2 aI2 rI2 ε30 - rI4 ε30 -
 2 aI3 rI2 ε30 + aI2 rI3 ε30 +
 2 aI2 rI2 ε31 - rI4 ε31 -
 2 aI3 rI2 ε31 + aI2 rI3 ε31 +
 2 aI2 rI2 ε32 - rI4 ε32 -
 2 aI3 rI2 ε32 + aI2 rI3 ε32 +
 2 aI2 rI2 ε33 - rI4 ε33 -
 2 aI3 rI2 ε33 + aI2 rI3 ε33 +
 2 aI2 rI2 ε34 - rI4 ε34 -
 2 aI3 rI2 ε34 + aI2 rI3 ε34 +
 2 aI2 rI2 ε35 - rI4 ε35 -
 2 aI3 rI2 ε35 + aI2 rI3 ε35 +
 2 aI2 rI2 ε36 - rI4 ε36 -
 2 aI3 rI2 ε36 + aI2 rI3 ε36 +
 2 aI2 rI2 ε37 - rI4 ε37 -
 2 aI3 rI2 ε37 + aI2 rI3 ε37 +
 2 aI2 rI2 ε38 - rI4 ε38 -
 2 aI3 rI2 ε38 + aI2 rI3 ε38 +
 2 aI2 rI2 ε39 - rI4 ε39 -
 2 aI3 rI2 ε39 + aI2 rI3 ε39 +
 2 aI2 rI2 ε40 - rI4 ε40 -
 2 aI3 rI2 ε40 + aI2 rI3 ε40 +
 2 aI2 rI2 ε41 - rI4 ε41 -
 2 aI3 rI2 ε41 + aI2 rI3 ε41 +
 2 aI2 rI2 ε42 - rI4 ε42 -
 2 aI3 rI2 ε42 + aI2 rI3 ε42 +
 2 aI2 rI2 ε43 - rI4 ε43 -
 2 aI3 rI2 ε43 + aI2 rI3 ε43 +
 2 aI2 rI2 ε44 - rI4 ε44 -
 2 aI3 rI2 ε44 + aI2 rI3 ε44 +
 2 aI2 rI2 ε45 - rI4 ε45 -
 2 aI3 rI2 ε45 + aI2 rI3 ε45 +
 2 aI2 rI2 ε46 - rI4 ε46 -
 2 aI3 rI2 ε46 + aI2 rI3 ε46 +
 2 aI2 rI2 ε47 - rI4 ε47 -
 2 aI3 rI2 ε47 + aI2 rI3 ε47 +
 2 aI2 rI2 ε48 - rI4 ε48 -
 2 aI3 rI2 ε48 + aI2 rI3 ε48 +
 2 aI2 rI2 ε49 - rI4 ε49 -
 2 aI3 rI2 ε49 + aI2 rI3 ε49 +
 2 aI2 rI2 ε50 - rI4 ε50 -
 2 aI3 rI2 ε50 + aI2 rI3 ε50 +
 2 aI2 rI2 ε51 - rI4 ε51 -
 2 aI3 rI2 ε51 + aI2 rI3 ε51 +
 2 aI2 rI2 ε52 - rI4 ε52 -
 2 aI3 rI2 ε52 + aI2 rI3 ε52 +
 2 aI2 rI2 ε53 - rI4 ε53 -
 2 aI3 rI2 ε53 + aI2 rI3 ε53 +
 2 aI2 rI2 ε54 - rI4 ε54 -
 2 aI3 rI2 ε54 + aI2 rI3 ε54 +
 2 aI2 rI2 ε55 - rI4 ε55 -
 2 aI3 rI2 ε55 + aI2 rI3 ε55 +
 2 aI2 rI2 ε56 - rI4 ε56 -
 2 aI3 rI2 ε56 + aI2 rI3 ε56 +
 2 aI2 rI2 ε57 - rI4 ε57 -
 2 aI3 rI2 ε57 + aI2 rI3 ε57 +
 2 aI2 rI2 ε58 - rI4 ε58 -
 2 aI3 rI2 ε58 + aI2 rI3 ε58 +
 2 aI2 rI2 ε59 - rI4 ε59 -
 2 aI3 rI2 ε59 + aI2 rI3 ε59 +
 2 aI2 rI2 ε60 - rI4 ε60 -
 2 aI3 rI2 ε60 + aI2 rI3 ε60 +
 2 aI2 rI2 ε61 - rI4 ε61 -
 2 aI3 rI2 ε61 + aI2 rI3 ε61 +
 2 aI2 rI2 ε62 - rI4 ε62 -
 2 aI3 rI2 ε62 + aI2 rI3 ε62 +
 2 aI2 rI2 ε63 - rI4 ε63 -
 2 aI3 rI2 ε63 + aI2 rI3 ε63 +
 2 aI2 rI2 ε64 - rI4 ε64 -
 2 aI3 rI2 ε64 + aI2 rI3 ε64 +
 2 aI2 rI2 ε65 - rI4 ε65 -
 2 aI3 rI2 ε65 + aI2 rI3 ε65 +
 2 aI2 rI2 ε66 - rI4 ε66 -
 2 aI3 rI2 ε66 + aI2 rI3 ε66 +
 2 aI2 rI2 ε67 - rI4 ε67 -
 2 aI3 rI2 ε67 + aI2 rI3 ε67 +
 2 aI2 rI2 ε68 - rI4 ε68 -
 2 aI3 rI2 ε68 + aI2 rI3 ε68 +
 2 aI2 rI2 ε69 - rI4 ε69 -
 2 aI3 rI2 ε69 + aI2 rI3 ε69 +
 2 aI2 rI2 ε70 - rI4 ε70 -
 2 aI3 rI2 ε70 + aI2 rI3 ε70 +
 2 aI2 rI2 ε71 - rI4 ε71 -
 2 aI3 rI2 ε71 + aI2 rI3 ε71 +
 2 aI2 rI2 ε72 - rI4 ε72 -
 2 aI3 rI2 ε72 + aI2 rI3 ε72 +
 2 aI2 rI2 ε73 - rI4 ε73 -
 2 aI3 rI2 ε73 + aI2 rI3 ε73 +
 2 aI2 rI2 ε74 - rI4 ε74 -
 2 aI3 rI2 ε74 + aI2 rI3 ε74 +
 2 aI2 rI2 ε75 - rI4 ε75 -
 2 aI3 rI2 ε75 + aI2 rI3 ε75 +
 2 aI2 rI2 ε76 - rI4 ε76 -
 2 aI3 rI2 ε76 + aI2 rI3 ε76 +
 2 aI2 rI2 ε77 - rI4 ε77 -
 2 aI3 rI2 ε77 + aI2 rI3 ε77 +
 2 aI2 rI2 ε78 - rI4 ε78 -
 2 aI3 rI2 ε78 + aI2 rI3 ε78 +
 2 aI2 rI2 ε79 - rI4 ε79 -
 2 aI3 rI2 ε79 + aI2 rI3 ε79 +
 2 aI2 rI2 ε80 - rI4 ε80 -
 2 aI3 rI2 ε80 + aI2 rI3 ε80 +
 2 aI2 rI2 ε81 - rI4 ε81 -
 2 aI3 rI2 ε81 + aI2 rI3 ε81 +
 2 aI2 rI2 ε82 - rI4 ε82 -
 2 aI3 rI2 ε82 + aI2 rI3 ε82 +
 2 aI2 rI2 ε83 - rI4 ε83 -
 2 aI3 rI2 ε83 + aI2 rI3 ε83 +
 2 aI2 rI2 ε84 - rI4 ε84 -
 2 aI3 rI2 ε84 + aI2 rI3 ε84 +
 2 aI2 rI2 ε85 - rI4 ε85 -
 2 aI3 rI2 ε85 + aI2 rI3 ε85 +
 2 aI2 rI2 ε86 - rI4 ε86 -
 2 aI3 rI2 ε86 + aI2 rI3 ε86 +
 2 aI2 rI2 ε87 - rI4 ε87 -
 2 aI3 rI2 ε87 + aI2 rI3 ε87 +
 2 aI2 rI2 ε88 - rI4 ε88 -
 2 aI3 rI2 ε88 + aI2 rI3 ε88 +
 2 aI2 rI2 ε89 - rI4 ε89 -
 2 aI3 rI2 ε89 + aI2 rI3 ε89 +
 2 aI2 rI2 ε90 - rI4 ε90 -
 2 aI3 rI2 ε90 + aI2 rI3 ε90 +
 2 aI2 rI2 ε91 - rI4 ε91 -
 2 aI3 rI2 ε91 + aI2 rI3 ε91 +
 2 aI2 rI2 ε92 - rI4 ε92 -
 2 aI3 rI2 ε92 + aI2 rI3 ε92 +
 2 aI2 rI2 ε93 - rI4 ε93 -
 2 aI3 rI2 ε93 + aI2 rI3 ε93 +
 2 aI2 rI2 ε94 - rI4 ε94 -
 2 aI3 rI2 ε94 + aI2 rI3 ε94 +
 2 aI2 rI2 ε95 - rI4 ε95 -
 2 aI3 rI2 ε95 + aI2 rI3 ε95 +
 2 aI2 rI2 ε96 - rI4 ε96 -
 2 aI3 rI2 ε96 + aI2 rI3 ε96 +
 2 aI2 rI2 ε97 - rI4 ε97 -
 2 aI3 rI2 ε97 + aI2 rI3 ε97 +
 2 aI2 rI2 ε98 - rI4 ε98 -
 2 aI3 rI2 ε98 + aI2 rI3 ε98 +
 2 aI2 rI2 ε99 - rI4 ε99 -
 2 aI3 rI2 ε99 + aI2 rI3 ε99 +
 2 aI2 rI2 ε100 - rI4 ε100 -
 2 aI3 rI2 ε100 + aI2 rI3 ε100 +
 2 aI2 rI2 ε101 - rI4 ε101 -
 2 aI3 rI2 ε101 + aI2 rI3 ε101 +
 2 aI2 rI2 ε1
```

```


$$\frac{a I^2 (a I^2 + (-2 + r I) r I) (-1 + \varepsilon^2) \cos[\theta I]^2 - i L^2 (a I^2 + (-2 + r I) r I) \cot[\theta I]^2}{i L^2 (a I^2 + (-2 + r I) r I) \cot[\theta I]^2} == 0$$

,

$$\varepsilon$$


$$] [[2]];$$



$$Ce =$$


$$p\theta I^2 + \cos[\theta I]^2 (a I^2 (1 - Ee^2) + i L^2 / \sin[\theta I]^2);$$


dynamicEquations =
{

$$r'[\tau] == \frac{pr[\tau] (a^2 - 2 r[\tau] + r[\tau]^2)}{a^2 \cos[\theta[\tau]]^2 + r[\tau]^2},$$


$$pr'[\tau] == \frac{\left(a^4 (-a Ee + L)^2 \cos[\theta[\tau]]^2 + a^4 (L^2 \cos[\theta[\tau]]^2 \cot[\theta[\tau]]^2 + p\theta[\tau]^2) r[\tau] + a^2 (-a^2 Ee^2 + 2 a Ee L - L^2 + 2 a Ee (a Ee + L) \cos[\theta[\tau]]^2 - 4 L^2 \cot[\theta[\tau]]^2 - 4 p\theta[\tau]^2) r[\tau]^2 + (4 a^2 Ee^2 - 8 a Ee L + 4 L^2 - 4 a^2 Ee^2 \cos[\theta[\tau]]^2 + 4 L^2 \cot[\theta[\tau]]^2 + 2 a^2 L^2 \cot[\theta[\tau]]^2 + 2 (2 + a^2) p\theta[\tau]^2) r[\tau]^3 + (-2 a^2 Ee^2 + 6 a Ee L - 4 L^2 + a^2 Ee^2 \cos[\theta[\tau]]^2 - 4 L^2 \cot[\theta[\tau]]^2 - 4 p\theta[\tau]^2) r[\tau]^4 + (L^2 \csc[\theta[\tau]]^2 + p\theta[\tau]^2) r[\tau]^5 - Ee^2 r[\tau]^6 + pr[\tau]^2 (a^2 - 2 r[\tau] + r[\tau]^2)^2 (a^2 \cos[\theta[\tau]]^2 - r[\tau]^2 + a^2 r[\tau] \sin[\theta[\tau]]^2)\right) / ((a^2 \cos[\theta[\tau]]^2 + r[\tau]^2)^2 (a^2 - 2 r[\tau] + r[\tau]^2)^2),$$


$$\phi'[\tau] == \frac{(a^2 L \cot[\theta[\tau]]^2 + 2 (a Ee - L - L \cot[\theta[\tau]]^2) r[\tau] + L \csc[\theta[\tau]]^2 r[\tau]^2) / ((a^2 \cos[\theta[\tau]]^2 + r[\tau]^2) (a^2 - 2 r[\tau] + r[\tau]^2)),$$


$$\theta'[\tau] == \frac{p\theta[\tau]}{a^2 \cos[\theta[\tau]]^2 + r[\tau]^2},$$


$$p\theta'[\tau] ==
\left( (2 a^2 \cos[\theta[\tau]] (\left(Ce + a^2 (-1 + Ee^2) \cos[\theta[\tau]]^2 - L^2 \cot[\theta[\tau]]^2\right) (a^2 - 2 r[\tau] + r[\tau]^2) - (Ce + (-a Ee + L)^2 + r[\tau]^2) (a^2 - 2 r[\tau] + r[\tau]^2) + (a L - Ee (a^2 + r[\tau]^2))^2) \sin[\theta[\tau]]) / (a^2 - 2 r[\tau] + r[\tau]^2) - a^2 p\theta[\tau]^2 \sin[2 \theta[\tau]] - a^2 pr[\tau]^2 (a^2 - 2 r[\tau] + r[\tau]^2) \sin[2 \theta[\tau]] + (a^2 \cos[\theta[\tau]]^2 + r[\tau]^2) (2 L^2 \cot[\theta[\tau]] + 2 L^2 \cot[\theta[\tau]]^3 - a^2 (-1 + Ee^2) \sin[2 \theta[\tau]])\right) / \left(2 (a^2 \cos[\theta[\tau]]^2 + r[\tau]^2)^2\right)$$

};

initialConditions =
{

$$r[0] == rI,$$


$$pr[0] == 0,$$


$$\theta[0] == \thetaI,$$


$$p\theta[0] == p\thetaI,$$


$$\phi[0] == 0$$

}

```

```

};

Quiet[
HamiltonianSolve =
NDSolve[
{
  dynamicEquations,
  initialConditions

    } /. {a → aI, L → iL},
  {r, φ, θ, pr, pθ},
  {τ, θ, pT},
  Method → {EventLocator, "Event" → (r[τ] - 1.02 holeSize)}
];
];
];

domain = (r /. HamiltonianSolve[[1, 1]])["Domain"];
{begin, end} = domain[[1]];
planetHasPlunged =
Abs[
(r[end] /. HamiltonianSolve)[[1]] - holeSize
] ≤ 0.05 holeSize;

startPlot =
If[
(end - tailLength) ≤ 0
,
0
,
(end - tailLength)
];
];

If[
zoomManual == False,
initialOuterRadius = (r[end] /. HamiltonianSolve)[[1]];
frameCantidate =
1.05
If[
initialOuterRadius > rI
,
initialOuterRadius
,
rI
];
];
];

```

```

frame =
If[
  frameCandidate > frame
,
  frameCandidate
,
  frame
];
{};

planetPosition =
{
  r[end] Sin[θ[end]] Cos[φ[end]],
  r[end] Sin[θ[end]] Sin[φ[end]],
  r[end] Cos[θ[end]]
} /. HamiltonianSolve;

orbitPlot =
ParametricPlot3D[
{
  r[τ] Sin[θ[τ]] Cos[φ[τ]],
  r[τ] Sin[θ[τ]] Sin[φ[τ]],
  r[τ] Cos[θ[τ]]
} /. HamiltonianSolve,
{τ, startPlot, end},
PlotRange →
{
  {-frame, frame},
  {-frame, frame},
  {-frame, frame}
},
PerformanceGoal → "Speed",
PlotPoints → 200,
MaxRecursion → 8, (*
  ViewPoint→rightViewPoint, *)
SphericalRegion → True,
Mesh → 4,
Ticks → Automatic
];

holeSize = 1 + Sqrt[1 - aI^2] ;

planetSize = 0.02 frame;

If[
planetHasPlunged
,
adjustedPlanetSize = 0;
,
adjustedPlanetSize = planetSize;
];
}

```

```

planetGraphic =
Graphics3D[{Green, Sphere[planetPosition, adjustedPlanetSize]}];

noReturnHorizon =
Graphics3D[{Black, Sphere[{0, 0, 0}, holeSize]}];

outerErgosphereLimit =
Graphics3D[
{Black,
Opacity[0.2],
Scale[
Sphere[], {2, 2, holeSize},
{0, 0, 0}
]}
];

```



```

rightImage =
Show[
orbitPlot,
noReturnHorizon,
outerErgosphereLimit,
planetGraphic,

Graphics3D[Text[StringForm["energy = ``", Ee], {1.5 frame, 0, -1.1 frame}]],

Graphics3D[
Text[StringForm["Carter Q = ``", Chop[Ce]], {1.5 frame, 0, -1.3 frame}]],

ViewPoint → rightViewPoint,
ImageSize → {400, 400}
];

```



```

leftImage =
Show[
orbitPlot,
noReturnHorizon,
outerErgosphereLimit,
planetGraphic,

Graphics3D[Text[StringForm["energy = ``", Ee], {1.5 frame, 0, -1.1 frame}]],

Graphics3D[
Text[StringForm["Carter Q = ``", Chop[Ce]], {1.5 frame, 0, -1.3 frame}]],

ViewPoint → leftViewPoint,
ImageSize → {400, 400}
]

```

```

,
{
{pT, tailLength, "time"}, 150, 1200,
ImageSize → Tiny,
AnimationRate → 3,
DisplayAllSteps → False,
DefaultDuration → 15,
ControlPlacement → Left
},
Delimiter,
{
{aI, 0.99, "spin rate"}, 0, 0.99, .01,
Appearance → "Labeled",
ImageSize → Tiny,
ControlPlacement → Left
},
Delimiter,
{
{rI, 4, "radius"}, 2.1, 30, .01,
Appearance → "Labeled",
ImageSize → Tiny,
ControlPlacement → Left
},
{
{iL, 2, "L"}, -4.5, 4.5, .01,
Appearance → "Labeled",
ImageSize → Tiny,
ControlPlacement → Left
},
{
{eI, π/3, Subscript[θ, I]}, π/7, 6π/7, π/210,
Appearance → "Labeled",
ImageSize → Tiny,
ControlPlacement → Left
},
{
{pθI, 0.76, Subscript[p, Subscript[θ, I]]}, -3, 3, .01,
Appearance → "Labeled",
ImageSize → Tiny,
ControlPlacement → Left
},
Delimiter,
{
{tailLength, 1200, "tail"}, 150, 1500,
}
```

```

ControlPlacement → Left,
ImageSize → Tiny
} ,

{
{frame, 4.5, "zoom"}, 2.5, 100, .01,
Appearance → "Labeled",
ImageSize → Tiny,
Enabled → zoomManual,
ControlPlacement → Left
} ,

{
{zoomManual, False, ""},
{False → "auto", True → "manual"},
ControlType → RadioButton,
ControlPlacement → Left
} ,

Delimiter,

{
{slidersEnabled, True, ""},
{False → "orbit preset"},
ControlType → Setter,
ImageSize → Tiny,
ControlPlacement → Left
} ,

{{presetValues, {1200, 0.99, 4, 2, π/3, 0.767851, 4.5, 1200, False}, ""},
{
{300, 0.9, 4, 2.148, 1.037, 0, 4.2, 350, False} →
Style["closed orbit", 10],
{1200, 0.99, 4, 2, π/3, 0.767851, 4.5, 1200, False} →
Style["constant radius orbit", 10],
{150, 0.0, 10, 3.5, π/2, 0, 4.5, 350, False} →
Style["spiral capture orbit", 10],
{150, 0.0, 4, 3.99999, π/2, 0, 4.5, 350, False} →
Style["unstable circular orbit capture", 10],
{100, 0.0, 4, 4.00001, π/2, 0, 4.5, 350, False} →
Style["unstable circular orbit escape", 10],
{330, 0.99, 25, 2.427, π/2, 0, 25, 330, False} →
Style["equatorial (1,1,1) zoom and whirl orbit"],
{150, 0.9, 4, -4.5, π/2, 0, 4.2, 350, False} →
Style["orbit reverse and capture", 10],
{150, 0.99, 10, 1.05769, π/2, 2.89, 4, 150, True} →
Style["3D zoom and whirl orbit", 10]
},
ControlType → PopupMenu,

```

```

ControlPlacement -> Left,
ImageSize -> Small
}},

Style[
"",
Bold, Small
],

SynchronousUpdating -> False,
SaveDefinitions -> True,
TrackedSymbols -> Manipulate,
AutorunSequencing -> {1, 2, 3, 4, 6, 7}
]

```



